



Goa University
P.O. Goa University, Taleigao Plateau, Goa 403 206, India
Syllabus of B.Sc. / B.A. (Mathematics) Programme

Approved by the Board of Studies in 2006 / 2010(Revised in 2012*)

A brief description of the course.

- **Purpose** : To train students for PG study of mathematics and to go for specialisation or career where mathematics is need as one subject.
To make available facility of taking Mathematics with Social science subjects.
- **Prerequisites**: 10 + 2 (Science) level mathematics
- **Credits** (theory, tutorials,) NA
- **Number of semesters, how the courses are distributed**: 6 semesters (2 per year)
(details given below)
- **Dissertation** : Project work at semester V and VI
- **Field work, etc.** NA

B. Sc./ B.A. (Mathematics) List of Courses

In the following tables, L refers to lectures, T to tutorials and P to practicals. Description of a course appears on the page number listed in the tables.

Compulsory Courses

Course Number and Name	L-T-P (hours/ week)	Credits	Page number
First Year B.Sc. Semester I			
Paper-101 : Calculus of one variable	60-0-0		5,6
Paper- 102 : Analytical Geometry	60-0-0		7
First Year B.Sc. Semester II			
Paper 201: Discrete Mathematics	60-0-0		8
Paper 202 : Probability & Statistics	60-0-0		9
Second Year B.Sc. Semester III			
Paper 301: Numerical Methods	36-0-24		10,11
Paper 302: Calculus of Two variables	60-0-0		12
Second Year B.Sc. Semester IV			
Paper 401: Matrix Algebra	36-0-24		13
Paper 402 : Differential Equations I	60-0-0		14
Third Year B.Sc. Semester V			
Paper 501: Analysis I	60-0-0		15
Paper 502: Algebra	60-0-0		16
Paper 503: Analysis II	60-0-0		17
Third Year B.Sc. Semester VI			
Paper 601: Linear Algebra	60-0-0		21
Paper 602: Metric Spaces	60-0-0		22
Paper 603: Complex Analysis	60-0-0		23

Compulsory Courses

Course Number and Name	L-T-P (hours/ week)	Credits	Page number
Third Year B.Sc. Semester V			
Paper 504: Vector Calculus	60-0-0		18
Paper 505: Number Theory	60-0-0		19
Paper 506: Operations Research I	60-0-0		20
Third Year B.Sc. Semester VI			
Paper 604: Analysis III	60-0-0		24
Paper 605: Differential Equations II	60-0-0		25
Paper 606: Operations Research II	60-0-0		26

Syllabus of the B. Sc. /B.A. Mathematics Curriculum

[Effective from academic year 2013-14]

First Year B.Sc.

Semester I: Paper 101 : Calculus of one variable

Paper 102 : Analytical Geometry

Semester II: Paper 201: Discrete Mathematics

Paper 202 : Probability & Statistics

Second Year B.Sc.

Semester III: Paper 301: Numerical Methods

Paper 302: Calculus of Two variables

Semester IV: Paper 401: Matrix Algebra

Paper 402: Differential Equations I

Third Year B.Sc.

Semester V: Compulsory Papers

Paper 501: Analysis I

Paper 502: Algebra

Paper 503: Analysis II

Optional Papers

Paper 504: Vector Calculus

Paper 505: Number Theory

Paper 506: Operations Research I

Semester VI: Compulsory Papers

Paper 601: Linear Algebra

Paper 602: Metric Spaces

Paper 603: Complex Analysis

Optional Papers

Paper 604: Analysis III

Paper 605: Differential Equations II

Paper 606: Operations Research II

Semester-I

Paper 101: CALCULUS OF ONE VARIABLE

1. FUNCTIONS AND GRAPHS.

[10 hrs ; 16 marks]

Prerequisites: Real Numbers, bounded sets. Definitions: Function, domain and range; One-one and onto functions. Examples. Graphical representation of functions. Polynomial and Rational functions. Power function: $y = x^\alpha$, where α is a real number ($x > 0$), General exponential function: $y = a^x$, where a is a positive number not equal to unity. Logarithmic function: $y = \log_a x$, where a is a positive number not equal to unity. Trigonometric functions: $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$ and $\csc x$. Inverse trigonometric functions: $\arcsin x$, $\arccos x$, $\arctan x$, $\text{arccot} x$, $\text{arcsec} x$ and $\text{arccosec} x$. Absolute value function ($|\cdot|$) Properties of the absolute Value function. Greatest integer function[]. Definitions of 'sup' and 'inf' of a non-empty subset S of IR .Theorems on 'sup' and 'inf'. Axiom of Lub (sup) .

Reference: Chapter 2 in [1] and/or Chapter 1 & 3 in [2] . Also [4]

2. LIMIT AND CONTINUITY.

[18 hrs ;24 marks]

Limit, left limit and right limit. Theorems:

$$(a) \lim_{x \rightarrow c} (f \pm g)(x) = \lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x) .$$

$$(b) \lim_{x \rightarrow c} (fg)(x) = \lim_{x \rightarrow c} f(x) \cdot \lim_{x \rightarrow c} g(x) .$$

$$(c) \lim_{x \rightarrow c} \left(\frac{f}{g}\right)(x) = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} \text{ provided } \lim_{x \rightarrow c} g(x) \neq 0 .$$

Limit of a function. Definition of 'lim f(x) as x -> infinity.' Uniqueness of limit of a Function. Continuity at a point, continuity in an interval, types of discontinuities. Theorems on continuity: (a) If a function is continuous on a closed interval, then it attains its bounds at least once in it. (b) If a function f is continuous at an interior point c of an interval and $f(c) \neq 0$ then f keeps the same sign of f(c) in a neighbourhood of c. (c) If a function f is continuous on a closed & bounded interval [a, b], and $f(a)f(b) < 0$, then there exists at least one point $c \in (a,b)$ such that $f(c) = 0$. (d) Intermediate value theorem.(e) fixed point theorem.

Reference: Chapter 5 in [3] . Also Chapter 3 in [4]

3. THE DERIVATIVE

[20 lectures ; 24 marks]

Drivability (Differentiability) at a point, Drivability in an interval, increasing and decreasing functions, Sign of the derivative. Higher order derivatives. Theorems: (a) A function which is derivable at a point is necessarily continuous at that point. (b) If f is derivable at c and $f(c) \neq 0$, then $1/f$ is also derivable at c. (c) Darboux's theorem. (d) Intermediate value theorem for derivatives.(e) Rolle's theorem. (f) Lagranges mean value theorem.(g) Cauchy's

mean value theorem.(h) Taylor's theorem.(i) Maclaurin's theorem. Increasing and decreasing functions.

Reference: Chapter 6 in [3]. Also Chapter 4 in [4]

4. APPLICATION OF TAYLOR'S THEOREM

[12 lectures; 16 marks]

Approximations. Extreme Values, Investigation of the points of Maximum and Minimum Values. Indeterminate forms, $0/0$ form, ∞/∞ form, Problems. Theorems:

(a) If $f(c)$ is an extreme value of a function f , then $f'(c)$, in case it exists, is zero. (b) If c is an interior point of the domain of a function f and $f'(c) = 0$, then the function has a maxima or a minima at c according as $f''(c)$ is negative or positive. (c) If f, g be two

functions such that (i) $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ and (ii) $f'(a), g'(a)$ exist and $g'(a) \neq 0$

then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}$. (d) L'Hopital's Rule for $0/0$ form. (e) If f, g be two functions such that

(i) $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} g(x) = 0$ and (ii) $f'(a), g'(a)$ exist and $g'(x) \neq 0$ for all $x > 0$ except

possibly at ∞ , and (iii) $\lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$ (f) L'Hopital's Rule for ∞/∞

form. Point of inflexion

Reference: Chapter 7 in [3]. Also Chapter 4 and 7 in [4]

References

- [1] Shanti Narayan, **Differential Calculus**, S.Chand & company (pvt)Ltd. 1988.
- [2] N.Piskunov, **Differential and Integral Calculus**, Vol.I, Translated from the Russian by George Yankovsky, CBS publishers & distributors, 4596/1A, 11 Darya Ganj, New Delhi-110 002.
- [3] S.C.Malik and Savita Arora, **Mathematical Analysis**, second edition, Wiley Eastern Ltd, 1994.
- [4] Tom Apostol, **Calculus Vol. I**, Second Edition, Wiley Students Edition, India, 2012

Paper 102 : ANALYTICAL GEOMETRY

1. Analytic Geometry of two Variables. [10 lectures; 20 marks]

General Equation of Second Degree. Equation $ax + 2hxy + by + 2gx + 2fy + c = 0$
Transformation of Co-ordinates. Change of Origin and Rotation of Axes. To show that the general second degree equation represents. (a) Ellipse if $h^2 < ab$. (b) Parabola if $h^2 = ab$. (c) Hyperbola if $h^2 > ab$. (d) Circle if $a = b$ and $h = 0$. (e) Rectangular Hyperbola if $a + b = 0$. (f) Two straight lines if $\Delta = 0$. (g) Two parallel straight lines if $\Delta = 0$ and $h^2 = ab$,

$$\text{where } \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}.$$

2. Conic sections. [20 lectures ; 30 marks]

Standard equations of conics using focus-directrix property. Parametric equations of standard conics. Tangent at a point (x_1, y_1) . Tangents in terms of slope. Tangent in terms of parametric co-ordinations. Condition of tangency. Properties of i) Parabola ii) Ellipse and iii) Hyperbola as listed in **Annexure 1**. Center of a Conic, Central Conic. Tangents and Normals. Pole & Polar with respect to conic.

3. Three Dimensional Geometry: Prerequisites: [20 lectures; 20 marks]

Direction Cosines, direction ratios. Equations of lines, planes, intersection of two planes, symmetric forms of equation, lines perpendicular to planes, angles between two lines and between a line and a plane. Projection of a line on a plane. Sphere: Intersection of a sphere by planes, intersection of two spheres

4. Central conicoids: [10 lectures; 10 marks]

Shapes, ellipsoids, hyperboloid of one sheet, two sheets. Intersection of a conicoid and a line. Cone and right cylinder. Standard equations

References

- [1] S. L. Loney; The Elements of Co-ordinate Geometry part I Cartesian Coordinates; subject publications 1990.
- [2] P.K.Jain, Khalil Ahmed: Textbook of Analytical Geometry of three Dimensions, second edition, Wiley, Eastern Limited, 1991.
- [3] M.L. Khanna: Solid Geometry; Jai Prakash Nath and Co.1988.
- [4] Tom Apostol , **Calculus Vol. I** , Second Edition, Wiley Students Edition , India, 2012

Semester II

Paper 201: Discrete Mathematical Structures.

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|--|-------------------------|
| 1. Propositional Calculus (Chapter 1. Last section only) | [5 lectures; 8 marks] |
| 2. Graphs (Chapter 5.) | [12 lectures; 16 marks] |
| 3. Trees (Chapter 6.) | [12 lectures; 16 marks] |
| 4. Discrete Numeric Functions (Chapter 9.) | [10 lectures; 12 marks] |
| 5. Recurrence Relation and Recursive Algorithms. (Chapter 10.) | [10 lectures; 12 marks] |
| 7. Boolean Algebra (Chapter 12.) | [11 lectures; 16 marks] |

Principal Text :

C.L.Liu: Elements of Discrete Mathematics,
McGraw-Hill International Editions, 1996.

References:

1. Kolman Busby and Ross: Discrete Mathematical Structures, PHI.
2. Rosen: Discrete Mathematics and Application, TMH
3. Schaum Series : 2000 Problems in Discrete Mathematics.
4. Schaum Series : Discrete Mathematical Structures.
5. Narsingh Deo : Graph Theory with Application to Computer Science.

Paper 202: Probability & Statistics*

Review of Probability and Random experiments. Theorems of total and compound probability. Total probability for n events (Statement only) Bayes' theorem, application problems.

[10 lectures; 10 marks]

Random Variables (Discrete and Continuous)- Probability, Distribution-Probability Density Functions- Mathematical Expectation- Functions of Random Variables. Joint Probability distribution. Marginal distribution function. Mathematical expectations and Generating Functions. Characteristic function.

[15 lectures; 15 marks]

Binomial and Poisson Distribution- Moments and Moment Generating Functions of these distribution and their simple properties- Recurrence Relations for moments of the Binomial and Poisson distributions. Fitting of Binomial and Poisson Distribution. Normal Distribution and its properties. Statement of the Lindberg-levy Central Limit Theorem.

[15 lectures; 20 marks]

Sampling and Large Sample Tests. Z test and Student's t test . the significance of sample mean, difference between two sample means, Variances. Snedecor's 'F' Distributions

Chi- square Distribution. Applications of the Chi-square Distribution-Tests of Goodness Fit-Contingences Tables.

[20 lectures; 35 marks]

Principal Text:

S.C.Gupta & V.K .Kapur – Fundamentals of Mathematical statistics (S.Chand)

Articles : 4.6 - 4.9 , 5.1 – 5.5.5 , 6.1 – 6.10 (omit 6.10.1-6.11.2), 6.12, 6.13, 6.17, (Omit 6. 12.1 - 6.12.3), 7.0 – 7.2.8 , 7.2.11,7.3.9 -7.3.10, 8.2 -8.2.14, 8.10, 8.10.2 (omit 8.2.6)

Chapter 12, Chapter 13 upto 13.8 {Omit 13.3.1,13.3.2 }

Chapter 14 upto 14.2.10, 14.5- 14.5.5.

Reference Text

- 1) Tom Apostol , **Calculus Vol. II** , Second Edition, Wiley Students Edition , India, 2012.
[Chapters 13 & 14]
- 2) Hogg and Craig: Mathematical Statistics
- 3) S.P.Gupta : Statistical Methods

Semester III

Paper 301 : Numerical Methods

Elementary Error Analysis :

[4 lectures; 6 marks]

Introduction. Numbers: Exact and Approximate. Significant digits. Errors: Absolute, Relative and Percentage errors. Examples

Calculus of Finite differences:

[6 lectures; 10 marks]

Operators Δ , ∇ & E . Difference Tables. Properties of Δ , ∇ , & E . Fundamental Theorem of Difference Calculus. Expression of any value of a function in terms of leading term and leading differences of a difference table. Method of separation of symbols.

Interpolation and Extrapolation:

[6 lectures; 12 marks]

.Newton's forward and backward interpolation formulae. Lagrange's Interpolation formula
.Newton's Divided Difference formula.
Examples based on the above formulae.

Numerical Differentiation and Integration:

[9 lectures; 14 marks]

Differentiation formulae for equidistant arguments. Examples. General quadrature formula for equidistant ordinates (Newton –Cotes Formula Or Gauss Legendre quadrature formulae). Trapezoidal rule and its Geometrical interpretation. Simpson's one third rule, Simpson's three-eighth rule. Weddle's rule (Only Statements)

Solution of Algebraic and transcendental Equations:

[6 lectures; 10 marks]

Introduction. Method of Bisection, Regula-Falsi Method, Newton-Raphson Method and their Computation scheme. Special Cases of Newton-Raphson Method like finding q^{th} root of a positive real number 'd' and finding reciprocal of a positive real number 'd' without using division.

Approximations:

[5 lectures; 8 marks]

The least square polynomial approximation method (Linear, quadratic, Exponential)

Practicals:

[24 lectures 20 marks]

- (1) Newton-Gregory Forward and Backward Interpolation Formulae.
- (2) Lagrange's Interpolation Formula for unequal intervals.
Newton's Divided difference formula for unequal intervals
- (3) Numerical Differentiation: Using Differentiation formulae for equidistant arguments
- (4) Numerical Integration: Trapezoidal rule, Simpson's one third rule and Weddle's rule
- (5) Solution of Algebraic and transcendental equations by the Bisection method.
- (6) Solution of Algebraic and transcendental equations by the Regula Falsi method and the Newton–Raphson method.
- (7) The least square polynomial approximation (Linear and quadratic)
- (8) The least square polynomial approximation (Exponential)

Note: Use Public domain soft wares like KASH and SCILAB is recommended.

References:

1. Introduction to Numerical Analysis : F.B.Hilderband
2. Introduction to Numerical Analysis . Atkinson
3. Numerical methods: Problems & Solutions . M. K. Jain, Iyengare Jain (Wiley)
- 4 Calculus of Finite differences & Numerical Analysis :
Gupta & Malik, Krishna Mandre, Prakashan Meerud
5. Introduction to Numerical Analysis : S.A.Mollah, New Central Book Agency Pvt. Ltd.
6. Tom Apostol , **Calculus Vol. II** , Second Edition, Wiley Students Edition , India, 2012.
[Chapter 15]

Paper 302 : CALCULUS OF TWO VARIABLES

1. FUNCTIONS OF TWO VARIABLES

[32 lectures; 44 marks]

Function of two variables, neighbourhood of a point, limit point, limit of a function, non-existence of limit, Algebra of limits, repeated limits, continuity, partial derivatives, differentiability, partial derivatives of higher order, change in the order of partial derivation, the derivation of a composite function (chain rule), change of variables, Extreme values, maxima and minima, sufficient condition for $f(x, y)$ to have an extreme value at (a, b) . Lagrange's Multipliers. Theorems: (a) Mean value theorem (b) Sufficient condition for continuity. (c) Sufficient condition for differentiability. (d) sufficient condition for the equality of f_{xy} and f_{yx} : Young's theorem and Schwarz's theorem. (e) Taylor's theorem; Maclaurin's expansion.

2. INTEGRATION ON R^2

[28 lectures; 36 marks]

Line integrals, plane curves, properties of line integrals. Double integrals, partition of a rectangle, integration over a rectangle.

Statements (only) of the following:

(a) Condition of integrability, integrals as a limit of sums, integrable functions (continuous function is integrable, bounded function with finite number of discontinuity is integrable etc.) (b) Repeated integrals, Calculation of double integral over a rectangle (reduction to repeated integrals). (c) Fubini's theorem. (d) Leibnitz's rule. (e) General Leibnitz's rule. (f) Two repeated integrals are equal. Double integrals over a region, Integrability over a bounded domain, reduction to iterated integrals. Change of variables and problems. Volume of a cylindrical solid by double integrals, volume enclosed by two surfaces, volume enclosed by closed surface.

References

- [1] N. Piskunov, **Differential and Integral Calculus**, Vol. II, Translated from the Russian by George Yankovsky, CBS publishers & distributors, 4596/1A, 11 Darya Ganj, New Delhi-110 002.
- [2] S.C. Malik and Savita Arora, **Mathematical Analysis**, second edition, Wiley Eastern Ltd, 1994, (Chapters 15, 16, 17)
- [3]. Tom Apostol, **Calculus Vol. II**, Second Edition, Wiley Students Edition, India, 2012.
[Chap. 8, 9 11]

Semester IV

Paper 401: MATRIX ALGEBRA *

1. Vectors and operations with vectors in \mathbb{R}^3 and generalization to \mathbb{R}^n . Linear combinations. Linear dependence and independence. Basis, Linear span and dimension. **[9 lectures; 15 marks]**
2. Elementary operations on a matrix: Elementary matrices. Effects of multiplying by these on a matrix. Equivalence of matrices: Row/column equivalence, Echelon forms. Normal form. **[9 lectures; 15 marks]**
3. Rank of a matrix: Definition using minors. Finding rank of a matrix using definition. (upto 3x3 only) Theorem: Elementary operations do not change the rank of a matrix. Finding the rank using echelon forms. Linear Independence of Row and Column Matrices. Definition of rank of a matrix using independence of Row or column vectors. Equivalence of two definitions of Rank. **[9 lectures; 15marks]**
4. Application of matrices. Solutions of a system of linear equations. Characteristic Values of a Matrix. Cayley-Hamilton Theorem. Diagonalisation of a matrix up to order 3 (when eigenvalues are distinct). **[9 lectures; 15 marks]**

PRACTICALS (Using C++)

[24 lectures; 20 marks]

1. Addition of matrices, Scalar multiplication and finding Determinant of a square matrix.
2. Multiplication of matrices.
3. Inverse of a square matrix using the formula $A^{-1} = \frac{1}{|A|} \times \text{Adjoint of } A$
4. Reduction of a matrix to the Echelon form and to find it's Rank.
5. Inverse of a matrix using row reduction.
6. Solution to a system of equations using Gaussian elimination.
7. Finding Eigen values and Eigen vectors of a matrix.
8. Diagonalization of a matrix and use it to find the powers of the matrix.

Note: Use Public domain soft wares like KASH and SCILAB is recommended.

References

- [1] K.B. Dutta; Matrices and Linear Algebra, Prentice Hall India
- [2] N. Ch. Iyengar: Matrices, Anamol Publications Pvt Ltd,1998.
- [3] Gilbert Strang: Introduction to Linear Algebra and its Applications, Thomson Books/cole. 1986.
- [4] Hadley G. Linear Algebra, Narosa PH. 1988.
- [5] Tom Apostol , **Calculus Vol. I & II** , Second Edition, Wiley Students Edition , India, 2012

Semester IV

Paper 402: Differential equations I

Review of basic concepts. Differential equation of the first order homogeneous, non-homogeneous, exact differential equations, conditions for exactness, integrating factors, integrating factors by inspection and rules for finding integrating factors, linear equations. Equations reducible to the linear form. Equations of first order, but not of first degree. Bernoulli's equation. Clairaut's form and equations reducible to it. Ricatti's equation. Applications. Modelling with differential equations.

[15 lectures; 20 marks]

Statement of sufficiency conditions for the existence and uniqueness of a solution of non-homogeneous differential equation $y'' + p(x)y' + q(x)y = r(x)$ together with the initial conditions $y(x_0) = y_0$; $y'(x_0) = y_1$.

Theorem: The dimension of the solution space of the homogeneous differential equation $y'' + p(x)y' + q(x)y = 0$ is two. General solution of the homogeneous equation. Characteristics Equation of a homogeneous differential equation with constant coefficients of order two and computation of linearly independent solutions. Wronskian. Use of known solution to obtain another linearly independent solution. Method of undetermined coefficients. Variation of parameters Formula. Extension of methods to n order. Applications of second order linear differential equation. (Models)

[25 lectures; 30 marks]

D-operator Method to solve linear differential equation with constant coefficients. $f(D)y = 0$. Solution for different types of roots. Inverse D-operator. Solution of $f(D)y = X$ where $X = \exp(kx)$, $\cos(kx)$, $\sin(kx)$, polynomials in x and their products.

$\{1/(D^2 + a^2)\} f(x)$, where $f(x) = \sin ax$, $\cos ax$.

[15 lectures; 20 marks]

Numerical solutions of ordinary differential equations. Euler's method. Taylor's series method. Picard's method of successive approximations.

[5 lectures; 10 marks]

References

- 1) Ordinary Differential Equations-G.F.Simmons (Tata McGraw Hill).
- 2) Advanced Engineering Maths- Kreyszig (wiley Eastern)
- 3) Differential Equation and Application –M.Braun (Narosa)
- 4) Ordinary Differential Equations – E.A.Coddington
- 5) Ordinary Differential Equations- Deo, Lakshmikantam & Raghvendra.
- 6) Numerical methods by Jain & Krishnamurthy.
- 7) Tom Apostol , **Calculus Vol. II** , Second Edition, Wiley Students Edition , India, 2012
[Chapter 6]

Semester V :

Paper 501: Analysis I

Sequences and series

1 Sequences in IR: Bounded Sequences, Algebra of sequences, Convergence of sequences, Sub sequences, monotone sequences, Cauchy sequences, Bolzano – Weierstrass theorem, Cauchy's General Principle of Convergence. Sequences in \mathbb{C} and \mathbb{R}^2 . **[15 lectures; 20 marks]**

2 Series (Real and Complex): Examples. Positive term series, Geometric Series, Power series, Alternating Series. Convergence of series. A necessary condition for convergence. Cauchy's General principle of Convergence, Absolute Convergence, Conditional Convergence, Comparison test for positive term series, Ratio test, Cauchy's root test, Leibnitz test for Alternating series. **[15 lectures; 20 marks]**

Sequences and series of functions:-

3 Examples of Sequences of real-valued functions, point wise and uniform convergence of sequences and of series of real valued and complex Valued functions defined on a subset of \mathbb{R} , Cauchy's Condition for uniform convergence of a sequence of functions, Continuity of the uniform limit function, uniform convergence. Properties of Boundedness, Integrability and Differentiability of the limit function. **[15lectures; 20 marks]**

4 Term by term integration and differentiation of series of functions from $\mathbb{R} \rightarrow \mathbb{R}$. Comparison test. Uniform convergence of Infinite series of functions. Cauchy's condition for uniform Convergence of series. Weierstrass's M-test for Uniform convergence. Dirichlet's test for uniform convergence. Uniform convergence and term by term integration and differentiation. Examples of non-uniformly convergent series that can be integrated term by term. **[15 lectures; 20 marks]**

Principal Text:

S. C. Malik & Arora.

Principles of Mathematical Analysis

References:

1. Goldberg R.R. Methods of Real Analysis
2. Prokter & Morrey Analysis
3. Walter Rudin Principles of Mathematical Analysis
4. T. Apostol Mathematical Analysis
5. R. Bartle Elements of Real Analysis
6. Das & Patnayak Principles of Mathematical Analysis
7. Tom Apostol , **Calculus Vol. I**, Second Edition, Wiley Students Edition , India, 2012

Paper 502: Algebra

1 Sets, Relations and mappings, equivalence relations, partitions. Binary operations and their properties. Divisibility in the set of integers.

Congruence modulo n , residue classes, addition and multiplication modulo n , Roots of unity.

[15 lectures; 20 marks]

2 Groups (definition and examples). Simple properties, subgroups, cyclic groups. Coset decomposition. Lagrange's theorem and its consequences. Fermat's and Euler's theorems. Homeomorphisms and isomorphisms of groups.

[20 lectures; 25 marks]

3 Normal subgroups, quotient groups. Fundamental theorem of group homomorphism. Permutation group, even and odd permutations, alternating group, Caley's theorem. Rings, Integral domain, Division rings and Fields.

[15 lectures; 20 marks]

4 Subring of a ring, characteristic of a ring, ideals. Homomorphism and isomorphism of rings, quotient rings. Fundamental theorem of ring homomorphism.

[10 lectures; 15 marks]

Principal Text :

J.B. Fraleigh : A First Course in Abstract Algebra

References

1. N.S. Gopalkrishnan: University Algebra
2. I.N. Herstein : Topics in Algebra.
3. J Gallian : Abstract algebra

Paper 503: Analysis II

1 Riemann Integral: Partition of an interval-properties of partitions- Upper and lower sums of a bounded real valued function over a closed interval-Riemann Integrability- Necessary and sufficient conditions. **[10 lectures; 15 marks]**

2 Riemann Integrals of Step, monotonic and continuous functions. Integrability of the absolute value, sums, scalar multiples of Riemann integrable Functions. Integrability of products, quotients and composition of functions. **[15 lectures; 20 marks]**

Theorems: (i) $\int_a^c f(x) dx + \int_c^b f(x) dx = \int_a^b f(x) dx$, $a \leq c \leq b$

$$(ii) \int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx .$$

$$(iii) \int c f(x) dx = c \int f(x) dx$$

$$(iv) \int_a^b f(x) dx = - \int_b^a f(x) dx$$

3 Continuity and differentiability of the integral as a function of the upper limit . Fundamental theorem of Calculus and the Mean Value Theorem for the Integral. **[10 lectures; 15 marks]**

4 Improper integrals of both types. Beta and gamma functions (basic definitions and simple properties.) **[25 lectures; 30 marks]**

Principal Text:

S.C. Malik and Arora : Mathematical Analysis

References:

1. R. Goldberg : Real analysis
2. Bhat R.D. : Mathematical Analysis II
3. R.B. Bartle : Introduction to Real analysis
4. Tom Apostol , **Calculus Vol. I** , Second Edition, Wiley Students Edition , India, 2012

Paper 504: Vector calculus *

1 Vector valued functions of a single variable. Their limits, continuity, derivatives and integrals. Space curves in \mathbb{R}^3 . Smooth and Regular curves. Arc-length parameter. Reparametrisation of curves. Tangent, Normal and Binormal vectors. Equations of tangent line and normal line. Torsion and Curvature. Serret- Frenet formula. Equations of fundamental planes.

[15 lectures; 20 marks]

2 Level surfaces. Scalar and vector fields. Vector differential operator. Gradient of scalar field and its properties. Directional derivatives. Curl and Divergence of vector field. Properties of curl and divergence. Irrotational and solenoidal vector fields. Identities on gradient, curl and divergence. Physical significance of gradient, curl and divergence. Laplacian operator.

[15 lectures; 20 marks]

3 Spherical and Cylindrical coordinates. Line integrals and its properties. Physical significance of line integrals. Independence of path. Problems on line integrals. Greens theorem(with proof) and its applications.

[15 lectures; 20 marks]

4 Surface and volume integrals. Stokes theorem(with proof) and its applications. Gauss divergence theorem(with proof)and its applications.

[15 lectures; 20 marks]

References:

1. Murray and Spiegel: Theory and Problems of Vector Analysis, SI (metric) edition, Schaum's Outline Series.
2. A.R. Vasishtha; Text book on Vectors, Krishna Prakashan Mandir (P) Ltd. 11, Shivaji Road, Meerut-250001(U.P),India.
3. J. N. Sharma and A.R.Vasishtha; Vector Calculus, Krishna Prakashan Media (P) Ltd.11,Shivaji Road, Meerut-250001(UP) India.
4. Shanti Narayan and J.N.Kapur; A text book of Vector Calculus, S. Chand & Company (Pvt.) Ltd, Ram Nagar, New Delhi-110055.
5. MD, Ali Ashraf and MD. Abdul Khaleq Hazra; Vector Analysis with Application, Third edition, Wiley Eastern limited.
6. Gosh and Maity; Vector Analysis, New Central book agency (P) LTD.
7. Tom Apostol , **Calculus Vol. I & II** , Second Edition, Wiley Students Edition , India, 2012

Paper 505 : Number Theory*

1) Divisibility: Divisibility Primes. Congruences, solution of congruences, Chinese Remainder Theorem Fermat Theorem, Wilson's theorem Congruences of degree 1

(15 lectures 20 marks)

2) Some Functions of Number Theory: , the function $\phi(n)$, Greatest integer function formula, the multiplication of Arithmetic functions,

(15 lectures 20 marks)

3 Quadratic Residues, Quadratic reciprocity, Jacobi symbol.

Some Diophantine Equations: the Equations $ax + by = c$, the equation $x^2 + y^2 = z^2$, the equation $x^4 + y^4 = z^4$, sum of Four and five squares.

(15 lectures 20 marks)

4 Simple continued fractions, Infinite continued fractions, Periodic continued fractions. Fibonacci numbers.

(15 lectures 20 marks)

Principal text: *Elementary Number Theory* By David Burton

Sixth edition Tata McGraw-Hill Edition

(Chapt. 2, 3 (section 3.1), 4 (sections 4.2, 4.4), 5 (Section 5.2 concerning Fermat's Theorem only, section 5.3), Chapt. 6 (section 6.1-6.3), Chap. 7 (Section 7.2, 7.4) Chapt. 9 (Section 9.1-9.3) Chapt. 12 (section 12.1, 12.2 (Theorem 12.3) Chapt. 15.2-15.3 Chapt. 14 (section 14.2)

References:

1. *An Introduction to the Theory of Numbers*
By I. Niven, H.S. Zuckerman and H. L. Montgomery,
Fifth edition, Wiley-India
2. *Elementary Number Theory with Applications* By Thomas Koshy
Second edition Elsevier India Pvt Ltd, 2005

Paper 506: Operations Research I *

1 Definition of standard form, formulation of LPP, convex set and extreme points of convex sets. (Only definitions) and examples . Graphical method (Only two variables).

[6 lectures; 10 marks]

2 Simplex Method: Theorems related to simplex method .and problems .Cases pertaining to existence of multiple solutions, unbounded and no feasible solution. Artificial techniques: Big M method and Two phase Simplex method

[24 lectures; 26 marks]

3 Duality, theorems on duality, linear programming problems with unrestricted variables. Dual simplex method. Revised simplex method.

[20 lectures; 24 marks]

4 Post – Optimal Analysis: Effects of change in the component of the cost vector and requirement vector, parameterization of the cost vector and requirement vector.

[10 lectures; 20 marks]

Principal Text :

Kantiswarup, P.K.Gupta & Man-Mahon: Operations Research

References:

- 1) H.Taha- Operations Research
- 2) Mittal & Goel—Operations Research
- 3) J.K.Sharma- Mathematical Methods in Operations Research

Paper 601: Linear Algebra

1 : Vector space [Definition and examples], subspaces, sum and direct sum of subspaces. Linear span, linear dependence, independence and their properties. Basis, existence theorem for basis, dimension of a vector space , finite dimensional vector spaces. Dimension of sum of subspaces. Existence of complementary subspace of a finite dimensional vector space. Quotient space and its dimension. **[15 lectures; 20 marks]**

2 Linear transformation, Fundamental theorem of Linear transformations . Vector space homomorphism, Matrix representation of linear transformation. Rank nullity theorem. **[15 lectures; 20 marks]**

3 Eigen values and eigen vectors of a linear transformation on a finite dimensional vector space. Eigen values of square matrix. Eigenspace. Cayley- Hamilton theorem. Diagonalisation of an $n \times n$ matrix over IR **[15 lectures; 20 marks]**

4 Inner products spaces. Cauchy- Schwarz inequality, orthogonal vectors, orthogonal complement, orthogonal sets and bases, Gram-schmidt orthogonalisation , Bessel's inequality, **[15 lectures; 20 marks]**

Principal Text:

Krishnamurty and others : Linear Algebra

References:

1. K.B.Datta : Matrix and Linear Algebra
2. K.Hoffman and R. Kunge : Linear Algebra
3. Schaum's series : Linear Algebra
4. S. Kumaresan: Linear Algebra :a geometric approach
5. Tom Apostol , **Calculus Vol. II** , Second Edition, Wiley Students Edition , India, 2012
[Chapters 1,2 and 4]
6. Gilbert Strang: Introduction to Linear Algebra and its Applications, Thomson Books, 4th edition

Paper 602 : Metric Spaces

1 Definition and examples of metric spaces.

Open ball and open sets, closed set as complement of an open set. Properties of closed sets and open sets. Limit points of a set, closure of a set, dense sets.

[15 lectures; 20 marks]

2 Subspace of a metric space. Convergence of a sequence in a metric space, Cauchy sequences. Continuous functions from a metric space X to a metric space Y (ϵ - δ definition), their characterization in terms of open sets, closed sets, closure and convergent sequences.

[20 lectures; 25 marks]

3 Complete metric space, completeness of a closed subspace of a complete metric space, Cantor's Intersection Theorem.

Contraction mapping, Fixed point theorem and its application to Picard's existence theorem for solution of a first order differential equation.

[15 lectures; 20 marks]

4 Connectedness in a metric space, Theorems on connectedness, Connected subsets of \mathbb{R} . Intermediate Value Theorem.

[10 lectures; 15 marks]

Principal Text :

E.T. Copson : Metric spaces, Cambridge University Press

References:

1. P.K. Jain and K. Ahmad : Metric spaces, Narosa Publishing House.
2. S. Kumaresan : Topology of Metric Spaces Narosa Publication House.

Paper 603: Complex Analysis.

1 Complex Numbers: Algebraic Properties of Complex Numbers, Modulus, Argand Diagram, Exponential Form and Polar Co-ordinates, Triangle inequality and Metric properties, Connectedness of regions. (Chapter 1.)_Analytic Functions: Complex and functions on Complex domain, Limits continuity of Complex valued function on a Complex domain, Differentiability and analytic Functions, Algebra of Differentiability, Cauchy-Riemann Equations, Sufficient condition for Differentiability, Harmonic Functions. (Chapter 2.)

[18 lectures; 24 marks]

2 Elementary Function: Exponential Function, Logarithmic Function and its Branches, Trigonometric Functions, Hyperbolic Functions. (Chapter 3.)

[14 lectures; 18 marks]

3 Contour Integration: Contours and Contour Integrals, Cauchy Goursat's theorem (with out proof), Simply Connected Domains, Cauchy's Integral Formula, Higher Derivatives of Analytic Functions, Liouville's Theorem, Fundamental Theorem of Algebra, Maximum Modulus Principle. (Chapter 4.) Series: Convergence of Series, Taylor Series, Laurent Series. (Chapter 5. First three sections on the above topics)

[20 lectures; 26 marks]

4 Residue Theory: Singularities of a Function, Poles and essential singularity, Residues at a singular point and its Computation, Residue Theorem

[8 lectures; 12 marks]

Principal Text

J.W.Brown and R.V.Churchil : Complex Variables and Applications
Sixth Edition, McGraw-Hill International Editions, 1996.

References:

1. A.R Shastri: Complex Analysis
2. Karunakaran: Complex analysis
3. Schaum's Series: Complex Analysis.
4. J.B. Conway; Functions of a Complex Variable.
5. S. Ponnuswamy: Complex Analysis. Narosa.
6. Sarason: Complex Analysis
7. Tom Apostol , **Calculus Vol. I** , Second Edition, Wiley Students Edition , India, 2012

[Chapter 9]

Paper 604: Analysis III

1: Weierstras's polynomial Approximation theorem. Power series in IR, their domain of Convergence, and Uniform convergence- term by term differentiation and integration of power series in IR.

Power series definitions of Exponential, Logarithmic and trigonometric functions, their properties.

[20 lectures; 30 marks]

2 Inner product : $(f, g) = \int f(x) g(x) dx$. Norm of f. Orthogonal system of functions. (Orthogonal and orthonormal sequences of functions).

[8 lectures; 10 marks]

3 Fourier series of real functions on $(-\pi, \pi)$ and $(0, \pi)$. Fourier coefficients, properties of Fourier coefficients, the Fourier series of a function relative to an orthonormal system. Bessel's inequality. Trigonometric Fourier series, Fourier series of odd & even function. Fourier series from power series.

[16 lectures; 20 marks]

4 Integration & differentiation of Fourier series at a point. Fourier theorem. Norm in $C[a, b]$. Cauchy-Schwartz inequality. Fourier Series of real functions on $(c, c+2l)$ Riemann-Lebesgue Lemma. Parseval's identity.

[16 lectures; 20 marks]

Principal Text :

S. C. Malik : Principles of Mathematical Analysis

References:

1. Golberg R.R. Methods of Real Analysis
2. R . Bartle Elements of Real Analysis
- 3 T. Apostol Mathematical Analysis
5. G.S.Sharma Engineering Mathematics
6. D. Somasundaram A first course in mathematical analysis
7. Tom Apostol , **Calculus Vol. I** , Second Edition, Wiley Students Edition , India, 2012

Paper 605 : Differential Equations II

1 Differential equations with Variable Coefficient which are analytic. Power series method. Legendre equation. Equation with regular singular points, exceptional cases. Bessel equation. Regular singular point at infinity. Gauss hypergeometric equation. Properties of Legendre Polynomial & Bessels functions. Laguerre equation, Tchebychev equations. Hermit equation. Euler equation.

[15 lectures; 20 marks]

2 System of 1st order differential equations. Conversion of nth order equation to 1st order system . Existence and uniqueness of solution (statement only). Methods of solution for Linear system . Homogeneous and non homogeneous equations with constant Coefficients . D operator method.

[20 lectures; 25 marks]

3 Laplace Transformation:-Introduction to Laplace Transformations –Laplace transformation of elementary functions- Laplace transformation of periodic functions- inverse Laplace transformation- Convolution Theorem. Solution of first order and second order linear differential equations with constant coefficients using Laplace transformation.

[20 lectures; 25 marks]

4 Numerical solution of differential equations. Multistep method . Predictor – corrector method Runge Kutta of order 2 and 4 . System of differential equations .

[5 lectures; 10 marks]

Principal Text:

G.F.Simmons : Differential equations with historical notes . TMH

References:

- 1) E.A.Coddington : An introduction to Ordinary Differential equations. PHI
- 2) S.G.Deo, Lakshmikantham, Raghavendra : A text book of Differential equations .
- 3) Schaum's Series : Lapalce Transform
- 4) S. Sastry: Numerical analysis
- 5) Tom Apostol , **Calculus Vol. II** , Second Edition, Wiley Students Edition , India, 2012

[Chapters 6]

Paper 606 : Operations Research II *

1 Queuing Theory:

Queuing system and its characteristics, Poisson Process, Exponential process, classification of queues – Transient and steady states, (M/M/C) : (∞ /FIFO) , (M/M/1) : (∞ /FIFO) . Queuing system.

[12 lectures; 16 marks]

2 PERT/CPM: Concepts of network, construction of network, Time estimates, CPM calculation, various floats, PERT calculations.

[11 lectures; 14 marks]

Transportation Problems: Mathematical formulation, condition for existence of feasible solution, rank of transportation matrix, Initial basic feasible solution by (i) NWC method (ii) Matrix-minima and (iii) VAM, Modi's method to find an optimal solution, balanced and unbalanced transportation problems.

[8 lectures; 10 marks]

3 Inventory Control:

Basis concepts of Inventory control, definition of inventory costs and other factors, Deterministic inventory problem (3 cases), Probabilistic inventory problems (discrete & continuous units)

[12 lectures; 16 marks]

4 Dynamic Programming;

Bellman's principle of optimality, recursive equation approach, characteristics of Dynamic programming, computational procedure in dynamic programming, multi-stage decision problems, solution of linear programming problem using dynamic programming

[10 lectures; 14 marks]

Assignment Problems: Mathematical formulation, Hungarian methods to solve assignment problems, balanced & unbalanced assignments problems

[7 lectures; 10 marks]

Principal Text:

Kantiswarup, P.K.Gupta & Man-Mohan : Operations Research

References :

1. H. .Taha- Operations Research
- 2 Mittal & Goyal- Operations Research

***Revised and New Syllabus**