<u>MATH-101: REAL ANALYSIS</u>

Number of Credits: 4

Prerequisites: Set Theory.

1) Real and Complex Number Systems: Finite sets, Natural Numbers and Induction Principle, Countable sets, Integers, Rational Numbers, Ordered Field, Complete Ordered Field, Real Numbers, Dedikind cuts, Complex Numbers.
Topology of Euclidean Spaces: Metric Spaces, Euclidean Spaces, Compact Sets, Perfect sets, Connected sets. **(15 hours)**

2) Continuity: Continuous Function, Continuity and Compactness, Continuity and Connectedness, Types of Discontinuities, Monotone Functions, Extended Real Number System, Infinite limits and Limits at Infinity. **(15 hours)**

3) Differentiation: Mean Value Theorem, Continuity of Derivatives, L'Hospital's Rule, Derivative of Higher order, Taylor's Theorem, Differentiation of Vector valued Functions**.** **(15 hours)**

4) The Riemann Stieltje's Integral: Definition and Existence of the Integral, The Integral as a limit of Sums, Integration and Differentiation, Integration of Vector Valued Functions, Functions of Bounded Variation, Properties of Integration, Rectifiable Curves. **(15 hours)**

References
[1] Walter Rudin, Principles of Mathematical Analysis, McGraw-Hill International Editions, 1976.
[2] N. L. Carothes, Real Analysis, Cambridge University Press, 2000
[3] Tom Apostol, Mathematical Analysis, Second Edition, Naroisa Publishing House, 1996.

MATH-102: ALGEBRA

Number of Credits: 4

Prerequisites:: Basic Group theory and basic Ring theory. This course is also a prerequisite for courses such as Commutative Algebra, Algebraic Geometry, Advanced Number Theory, and Cryptography.

1) Preliminaries: Basic Group Theory, Basic Ring theory and in particular the the Characteristic of an Integral Domain and polynomial Rings over fields and construction of fields using irreducible polynomials, and Basic Linear Algebra including basis and dimension of a Linear Spaces, and Linear maps.
Groups and subgroups: Definition and examples of groups. Subgroups. Cyclic groups. **(15 hours)**

2) Permutations, Cosets and Direct Products: Group of Permutations, Orbits, cycles and Alternating groups. Cosets and Theorem of Lagranges, Euler's Theorem, Wilson's Theorem, Direct products and Finitely generated abelian groups.
Homomorphism and Factor groups: Homomorphisms and Factor groups**. (15 hours)**

3) Advanced Group Theory: Isomorphism Theorems. Series of groups. Sylow Theorem. Applications of Sylow theorem. Free abelian groups.
Ring Theory. Rings and Fields: Rings. Fields. Integral domain. Fermats and Euler theorems. Field of quotients of an integral domain. **(15 hours)**

4) Ring of polynomials. Factorization of polynomials over a field.
Ideals and Factor Rings: Homomorphisms and Factor rings. Prime and maximal ideals. Factorization: Unique factorization domains. Euclidean domains, Gaussian integers, Euler's Theorem on expressing $p = a^2 + b^2$. **(15 hours)**

References

[1] J.B. Fraleigh, A First Course in Abstract Algebra, Seventh Edition, Pearson International, 2002.
[2] I. N Herstein, Topics in Algebra, Second Edition, Wiely Student Edition, 2006.

MATH-103: LINEAR ALGEBRA

Number of Credits: 4

Prerequisites: This course assumes basic knowledge in Linear Algebra such as definition of Vector spaces, Basis, Dimension of Vector Spaces, Linear Transformation and their Matrix Representation, Singular and Nonsingular linear Transformation, Algebra of Linear maps from a vector space to another and some elementary properties of Polynomial Ring over a Field.
This course is also a prerequisite for courses such as Several Variable Calculus, Differential Equations, Representation of Finite Groups, Field Theory, Differential Geometry, and Lie Group Theory.

1) Preliminaries: Recall of Basic Facts listed in the First Paragraph. (This has to be done with necessary details for absorbing the remaining part of the course.) Dual of a Vector Space: Definition of Functional, Dual of a Vector Space, Transpose of a Linear Transformation, The Matrix of the Transpose. **(15 hours)**

2) Elementary Canonical Forms: Characteristic Values, Annihilating Polynomial, Invariant Subspaces, Simultaneous Triangulation, Simultaneous Diagonalization, **(15 hours**)

3) Direct Sum Decomposition, Invariant Direct Sums, Primary Decomposition. Inner Product Spaces: Inner Product Spaces. **(15 hours)**

4) Linear Functional and Adjoints, Unitary and Normal Operators. Operators on Inner-Product Spaces: Forms on Inner-Product Spaces, Positive forms and other forms, Spectral Theory, Properties of Normal Operators. **(15 hours)**

References

[1] Kenneth Hoffman and Ray Kunze, Liner Algebra,Prentice Hall India, 1997.
[2] J.B. Fraleigh, A First Course in Abstract Algebra, Seventh Edition, Pearson International.
[3] I. N Herstein, Topics in Algebra, Second Edition, Wiely Student Edition, 200
(4)Hadley: Linear Algebra Springer Verlag, 1979.

MATH-104: TOPOLOGY

Number of Credits: 4

Prerequisites: This Course assumes basic knowledge of Real Numbers particularly it completeness, and Metric Space Theory. This course is a prerequisite for course in Algebraic Topology, Several Variable Calculus, Differential Geometry, Harmonic Analysis, Functional Analysis, The Theory Lie Groups, and Riemannian Geometry.

1) Topological Spaces and Continuous Functions: Definition of Topological Spaces and Continuous maps, Product Topology, Subspace Topology, Closed Sets, Limit points, Closure, Interior and Boundary, Quotient Topology. **(15 hours)**

2) Countability and Separation Axioms: First and Second Countable Spaces, Hausdorff Spaces, Regular and Normal Spaces, Urysohn's Lemma and Tietze's Extension Theorem, Urysohn Metrization Theorem**.** **(15 hours)**

3) Connectedness and Compactness: Connected Spaces, Connected Subsets of the Real Line, Path Connectedness, Locally connected and locally Path Connected Spaces, Components and Path Components, Compact Spaces, Compact Subspaces if Real Line, **(15 hours)**

4) Finite Product of Compact Spaces, Tychonoff Theorem, Compactness and Continuous maps, One-point Compactification.
Fundamental Group: Homotopy of Paths, The Fundamental Group, Covering Spaces, The Fundamental Group of The Circle. **(15 hours)**

References

[1] James R. Munkres, Topology, Second Edition, Pearson Education Asia, 2002.
[2] O.Y. Viro et al : Elementary Topology Problem Text book. AMS 2008
[3] Lipschutz S. General Topology, Schaum series, MG 1965

MATH-105: SEVERAL VARIABLES CALCULUS

Number of Credits: 4

Prerequisites: This Course assumes basic knowledge in Real Analysis, Linear Algebra. Knowledge of Integration of a Real Valued Function on a subset of R is desirable. This course is a prerequisite for a course on Differentiable Manifolds.

1) Differential Geometry, The Theory Lie Groups and a course on Riemannian Geometry.Derivative of Function of more than one Variable: Partial Derivative, Total Derivative of function of more than one Variable, Jacobian, Sufficient Condition for Differentiability, Mean Value Theorem, Higher Derivative, Condition for Equality of Mixed Partial Derivative, Taylor's Theorem. **(15 hours)**

2) Maximum Minimum: Critical Point, Maximum Minimum, Second Derivative Condition for Maximum/minimum, Conditional Optimum and Lagrange Multipliers. Inverse Function Theorem: Regular and Singular Points, Open Mapping Theorem, Inverse Function Theorem, Implicit Function Theorem. **(15 hours)**

3) Riemann Integration: Rectangles in Rn and Riemann sums over Rectangles, Upper and Lower Riemann Sums, Riemann Integral of a bounded Function, Algebra of Riemann Integrals, Sets of Jordan Measure Zero, Oscillation of a Function at a point, Integrability versus Points of Discontinuity of a Function, Fubini's Theorem. **(15 hours)**

4) Differential Forms: Tensor Product of Functionals, Alternating forms, Vector Fields, Differential Forms, Wedge of Differential Forms, Differential of a Differential Form, Non-Commutativity Wedge Product, @2 = 0, Simplexes, Chain Complexes, Boundary Operator, Stokes Theorem. **(15 hours)**

References

[1] Tom M Apostol, Mathematical Analysis, Addison Wesley Publishing Company, 1996.
[2] James Munkres, Analysis on Manifolds, Addison Wesley Publishing Company, 1991.
[3] Walter Rudin, Principles of Mathematical Analysis, International Student Edition.

MATH-106: COMPLEX ANALYSIS

Number of Credits: 4

Prerequisites: Real Analysis of Power Series, Complex numbers, Analytic functions, Basics of Contour Integration.

1) Elementary properties and examples of analytic functions: Power Series, Analytic Functions, analytic Functions as mappings. (Chapter III)
Complex Integration: Riemann Stieltje Integrals, **(15 hours)**

2) Power Series Representation
of Analytic Functions, Zeros of an Analytic Function, The Index of a Closed Curve, Cauchy's Theorem and Integral Formula, The Homotopic Version of Cauchy's Theorem and Simple Connectivity, Counting Zeros, Open Mapping Theorem.(Chapter IV)
(15 hours)

3) Singularities: Classification of Singularities, Residues. (Chapter V).
The maximum modulus theorem: The Maximum Principle Schwarz Lemma. (Chapter VI) **(15 hours)**

4) Compactness and convergence in the space of analytic functions: The Space of Continuous Functions C(G,), The Space of Analytic Functions, the Space of Meromorphic Functions, The Riemann Mapping Theorem, Weierstrass Factorization Theorem, Factorization of sine Function, The Gamma Function and the Riemann Zeta Function. (Chapter VII) **(15 hours)**

References

[1] J.B. Conway, Functions of One Complex Variable, Second Edition, Springer International Student Edition, 1978.
[2] Lars Alfors, Complex Analysis, Third Edition, McGraw-Hill International Editions, 1979.
[3] Brown and Chirchill, Complex Analysis with Applications,
[4] Walter Rudin, Reeal and Complex Analysis, International Student Edition,

MATH-107: NUMBER THEORY

Number of Credits: 4

Prerequisites:Basic Mathematics.

1) Divisibility; Congruences, Solutions of congruences, Congruences of degree 1, The function _(n); Quadratic residues, Quadratic reciprocity, **(15 hours)**

2) The Jacobi symbol;Greatest integer function, Arithmetic functions, The Moebius inversion formula, **(15 hours)**

3) The multiplication of arithmetic functions; Diophantine equations, The equation x2 + y2 = z2, The equation x4 + y4 = z2, Sum of four squares; **(15 hours)**

4) Simple continuedfractions, Infinite continued fractions, Periodic continued fractions; Fibonacci numbers;Partitions of numbers, Generating functions, Theorems of Euler, A theorem of Jacobi, Special cases of Jacobi's identity **(15 hours)**

References

[1] I. Niven, H.S. Zuckerman and H. L. Montgomery, An Introduction to the Theory of Numbers, Fifth edition, Wiley-India.
[2] G.H. Hardy and E.M. Wright, An Introduction to the Theory of Numbers, IV Edition Clarendon Press.
[3] David Burton, Elementary Number Theory, Sixth edition Tata McGraw-Hill Edition.
[4] Thomas Koshy, Elementary Number Theory with Applications. Second edition

MATH-108: METHODS OF APPLIED MATHEMATICS

Number of Credits: 4

Prerequisites: Real Analysis , Linear Algebra , Differential equations.

1) Review of series solutions:. Legendre's equations, Bessel's equations, Laguerre's equation, Hermite's equation, Chebyshev's equation and their solutions, Special functions associated with these equations and their properties, Generating functions (Ref [1]). **(15 hours)**

2) Fourier series: Generalized Fourier series, Fourier sine/cosine series, Pointwise and uniform convergence, differentiation and integration of Fourier series. (Chap 2, Reference [5].) **(15 hours)**

3) Fourier Transforms and its properties: Inversion Theorem, Convolution Theorem. Fourier Transform of L1(R)-functions, basic properties related to translation, dilation and linearity, Statement Fourier inversion, computation of Fourier transform of simple examples, Convolution, Fourier transform taking convolution into product . (Ref [2]). **(15 hours)**

4) Variational problems with fixed boundaries: Euler-Lagrange equations, Brachistochrone problem, Geodesics, Minimum surface of revolution, Elementary variational problems with moving boundaries, One-side variation, Isoperimetric problem, Canonical forms of Euler equations, some direct methods in calculus of variations (Ref .[3]) **(15 hours)**

References

[1] E.A. Coddington, An introduction to Ordinary Differential Equations, Prentice Hall India.
[2] K.Sankara Rao, Introduction to Partial Differential Equations, Prentice Hall of India, 1995.
[3] Lev Elsgolts, Introduction to the Calculus of Variations, MIR Publications.
[4] G.B.Arfken and H. Weber, Mathematical methods for Physicists Elsevier Publications.
[5] J.W.Brown and R.V.Churchill, Fourier series and Boundary Value Problems, McGraw Hill.
[6] R. Weinstock, Calculus of Variations, Dover Publication.
[7] I.M.Gelfand and S.V.Fomin, Calculus of variations, Dover Publication.

MATH-109: NUMBER THEORY II

Number of Credits: 4

Prerequisites: Some basic Complex Analysis. This course also will serve as Prerequisites to an advanced Course in Analytical Number Theory.

1) Recall of basic notions of Divisibility, Congruence, Arithmetical Functions, Quadratic Residues, Quadratic Reciprocity, Jacobi Symbol, Diophantine equations . Simple Continued Fractions and Fibonacci Numbers has to be done with emphasis on problem solving. **(15 hours)**
2) Arithmetical functions and Dirichlet multiplication. Averages of arithmetical functions. Some elementary theorems on distribution of prime numbers**.** **(15 hours)**
3) Characters of finite abelian groups. Dirichlets theorem on primes in arithmetic
4) Progression. Periodic arithmetical functions and Gauss sums **(15 hours)**
5) Primitive roots. Dirichlet series and Euler products. Basic Cryptology. **(15 hours)**

References

[1] T. M Apostol, *Introduction oto Analuytic Number Theory*, Narosa Publishing House.
[2] Heng Huat Chan, *Analytic Number Theory for Undergraduates,* (Monographs in Number Theory), World Scientific, **2009**.
[3] I. Niven, H.S. Zuckerman and H.L. Montgomery, *An Introduction to the Theory of Numbers*, Fifth edition, Wiley-India.
[4] David Burton, *Elementary Number Theory*, Sixth edition, Tata McGraw-Hill Edition.
[5] Thomas Koshy, *Elementary Number Theroy with Applications*, Second Edition, Elsevier India Pvt. Ltd., **2005**.

<u>MATH; 110: THEORY OF COMPUTATION</u>

Number of Credits: 4

<u>Prerequisites:</u> This Course assumes basic knowledge in Basic Set Theory. some familiarity with Discrete Mathematics is desirable.

1) Theory of Finite Automata: Definition and Examples, Designing Finite Automata, Regular Operations, Determinism, Equivalence of DFA and NFA, Regular Operations, Regular Expressions and their Equivalence, , Non-regular Languages**. (15 hours)**

2) Context Free Languages: Definition and Examples of Context free Grammars, Designing Context Free Grammars, Chomsky Normal Form, Pushdown Automata and it Equivalence with Context-free Grammars, Non-Context Free Languages, The Pumping Lemma. **(15 hours)**

3) Computability Theory: Turing Machines, Multi-tape Turing Machines, Nondeterministic Turing Machines, Equivalence of the models of Turing Machines, Hilbert's Problem, Decidability Problem of Regular and Context-free Languages, Halting Problem, The Diagonalization method, Turing Unrecognizable Language, Undecidability Problem in Language Theory, Mapping Reducibility. **(15 hours)**

4) Theory of Complexity: Time Complexity, Big and Small o's, Complexity Relationship among models, The Class P and the Class NP, P verses NP, NPCompleteness, The Cook Levin Theorem, The Vertex Cove Problem and the Hamiltonian Path Problems, Space Complexity, The Class L and NL, NL-Completeness, Searching Graph. **(15 hours)**

References

[1] Michael Sipser, Introduction to the Theory of Computation,Thomson Learning, 1997.

MATH-111: MATRIX GROUPS

Number of Credits: 4

Prerequisites: Several variable calculus, basic facts about complex numbers, linear algebra, basic group theory, basic analysis.

1) Matrices, Real and Complex Matrix Groups, Orthogonal Groups, **(15 hours)**
2) Topology of matrixgroups, Matrix exponentiation, Tangent space, **(15 hours)**
3) Structure of Lie algebras, **(15 hours)**
4) Representationsof Lie algebras, Root system. **(15 hours)**

References

[1] Kristopher Tapp, Matrix Groups for Undergraduates, American Mathematical Society, 2005.

[2] Jphn Stillwell, Naive Lie Theory, Springer Verlag, 2008

[3] A. Baker, Matrix Groups: An Introduction to Lie Theory, Springer Verlag, 2002.

[4] Karin Erdmann and Mark J. Wildon, Introduction to Lie Algebras, Springer Undergraduate Mathematics Series, Springer-Verlag. 2006.

MATH-112: DIFFERENCE EQUATIONS

Number of Credits: 4

Prerequisites: Real Analysis , Linear Algebra , Differential equations.

1) Linear Difference Equations. Equilibrium Points and Dynamics. Logistic equation. Methods of Undetermined Coefficients and Variation of Parameters. **(15 hours)**

2) Behaviorof Solutions. Nonlinear equations. Systems of Difference Equations**.** **(15 hours)**

3) Basic Theory and Applications.Higher Order equations. Stability theory. Z-Transforms and its applications. (**15 hours)**

4) Volterra Difference Equation of Convolution Type **.(15 hours)**

References

[1] S.N.Elaydi, An Introduction to Difference Equations, Springer Verlag. (Chapters 1 to 5)
[2] F.Chorlten, Differential and difference equations., von Nostqand.
[3] S.Goldberg , Introduction to Difference equations, Wiley Publication.
[4] V.Lakshmikantham and D.Trigiante, Theory of difference equations, Academic Press.
[5] K.Miller, Linear Difference e

MATH- 113: Mathematics for Finance

Number of Credits: 4

Prerequisites: Basic Real analysis

1) Introduction: A Simple Market Model. No arbitrage Principle. Risk and returns. Forward options . Call and Put it options. Risk-Free Assets**.** Time value of money and money market (Chapters 1 , 2 of Ref. [1]) **(15 hours)**

2) Risk Assets. Dynamics of stock prices. Binomial Tree and other models. Tree and other model. Discrete Time Market Models . (Chapters 3,4 of Ref[1]) **(15 hours)**

3) Portfolio Management. Securities. Capital asset Pricing model. Contracts Forward and Future. (Chapters 5,6 of Ref [1]) **(15 hours)**

4) Options. Types and bounds. Variable Interest Rates. Maturities and yields. (Chap. 7, 10 of Ref [1]) **(15 hours)**

References:

1. Mathematics for Finance: Marek Capinski and T. Zastawnik Springer Verlag 2003. Chapters 1-7:10
2. Intrest Rate Models Theory and Practice Damiano Brigo Fabio Mercurio, Springer
3. Risk analysis in Finance and Insurance Alexander Melinkov Chapman & Hall
4. Options ,Futures and other derivatives, J.C.Hall and S. Basu, 7th edition Pearson, Chenai

<u>MATH- 114 : Actuarial Science</u>

Number of Credits: 4

<u>Prerequisites:</u> Basic Real analysis

1. Basic concepts of actuarial science and insurance. Accumulated Value, Present Value Principals of compound interest: Normal and effective rates of interest and discount, force of interest and discount compound interest, accumulation factor, Annuities certain. Deferred annuities, annuities, annuities due, Redemption of Loans, Sinking Funds and Capital redemption assurance. **(15 hours)**

2. Life insurance: Insurance payable at the moment's of death and at the end of the year of death-level benefit insurance, endowment insurance, differed insurance and varying benefit insurances, recursions, commutation functions, Life annuities : Single payment, continuous life annuities, discrete life annuities, life annuities with monthly payments, commutation functions, varying annuities, recursions, complete annuities-immediate and apportion able annuities-due**.** **(15 hours)**

3. The Mortality tables. Functions and laws of mortality tables. Select ultimate and aggregate mortality tables. Functions other than yearly policy Values. Surrender values and paid up Values, Bonus Special policies. Joint life and last survivor statuses. **(15 hours)**

4. Models of Actuarial Science. **(15 hours)**

<u>Reference</u>

1 N./L.Bower, H.U.Gerber, J.C.Hickman, D.A.Jones and C.J.Nesbitt (1986), Actuarial Mathematics society of Actuaries, Itasca, IIIinos, USA Second Edition (1997)

2. Spurgeon E.T. (1972), Life Contingencies, Cambridge University Press.

3. Neill, A. (1977). Life Contingencies, Heinemann

4. M.A. Mackenzie, N.E.Sheppard, An Introduction to the Theoryof Life Contingencies, 1931.

5. P.Zima & R.L.Brown, Mathematics of Finance Schaum's Outline series

6. Elements of actuarial science Premiums, Mortality and valuation Federation of insurance institutes P.M. road, Mumbai.

7. S. Svoboda, Interest rates modeling, Macmillan, 2004.

MATH- 115: Numerical Analysis.

Number Of Credits: 4

Prerequisites: Basic Real Analysis.

1) Principles of floating point computations and rounding errors. Systems of Linear Equations: factorization methods, pivoting and scaling, residual error correction method. Iterative methods: Jacobi, Gauss-Seidel methods with convergence analysis, conjugate gradient methods**.** Eigen value problems: only implementation issues. **(15 hours)**

2) Nonlinear systems: Newton and Newton like methods and unconstrained optimization. Interpolation: review of Lagrange interpolation techniques, piecewise linear and cubic splines, error estimates. Approximation: uniform approximation by polynomials, data fitting and least squares approximation. **(15 hours)**

3) Numerical Integration: integration by interpolation, adaptive quadratures and Gauss methods. Initial Value Problems for Ordinary Differential Equations: Runge-Kutta methods, multi-step methods, predictor and corrector scheme, stability and convergence analysis. **(15 hours)**.
4) Two Point Boundary Value Problems : finite difference methods with convergence results.
   Lab. Component: Implementation of algorithms and exposure to public domain packages like LINPACK and ODEPACK. **(15 hours)**

References

1. K.E. Atkinson, An Introduction to Numerical Analysis, Wiley, 1989.

2. S.D. Conte and C. De Boor, Elementary Numerical Analysis %G–%@ An Algorithmic Approach, McGraw-Hill, 1981.

3. K. Eriksson, D. Estep, P. Hansbo and C. Johnson, Computational Differential Equations, Cambridge Univ. Press, Cambridge, 1996.

4.G.H. Golub and J.M. Ortega, Scientific Computing and Differential Equations: An Introduction to Numerical Methods, Academic Press, 1992.

5.J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, 2nd ed., Texts in Applied Mathematics, Vol. 12, Springer Verlag, New York, 1993.

MATH-201: DIFFERENTIAL EQUATIONS

Number of Credits: 4

Prerequisites: Real Analysis, Calculus of Several Variables and Ordinary Differential equations.

1) Review of ordinary differential equations: Existence and uniqueness of solutions to first order equations,systems and nth-order equations. (Ch. 5, 6 of[1]) Simultaneous differential equations of the first and first degree in three variables: Methods of solutions of dx/P = dy/Q = dz/R, Pfaffian differential forms and equations, Solutions of Pfaffian differential equations in three variables. (Chap 1, Sec 1-6, of [3]). **(15 hours)**

2) First order PDE's: Origin and classifications, Solutions of Linear and Nonlinear First order PDE's, Methods of Characteristics, Charpit's Methods, Jacobi's method. (Chap 2, Sec 1-13, of [3]).
Second Order Linear Partial Differential Equations: Origin, Linear equations with constant coefficients in two independence Variables. (Chap3, Sec.1 and4, of [1]). (**15 hours)**

3) Linear equations with variable coefficients-Classification: Reduction to Canonical Form (only for the case of two independent variables), One dimensional Wave equation-D' Alembert' solution, Wave equation-infinite string case.
Basic Properties of harmonic functions Laplace equation: Translational and rotational invariance of Laplace equation, **(15 hours)**

4) Boundary value problems. Uniqueness of solutions of Dirichlet and Neumann problems, Mean value theorem for harmonic functions, Maximum and minimum principle for harmonic functions,
Uniqueness and stability for Dirichlet problem, Heat equation- infinite rod case. ([2], [5]) Method of Separation of variables: Use of Integral transforms (Laplace and Fourier), Method of Greens function. ([2], [3], [5]). **(15 hours)**

References

[1] E.A. Coddington, An introduction to Ordinary Differential Equations, Prentice Hall India.
[2] T.Amarnath, An elementary course in Partial Differential Equations, Narosa Publishing company, 1997.
[3] I. Sneddon, Elements of Partial Differential Equations, McGrow Hill.
[4] F.John, Partial Differential equations, Springer Verlag Ltd.
[5] C.R. Chester, Techniques of Partial Differential Equations.
[6] R.Dennemeyer, Introduction to Partial Differential Equations and Boundary Value Problems, McGraw Hill.
[7] S.Kesavan, Topics in Functional Analysis and Applications, Wiley Eastern India.
[8] K.Sankara Rao, Introduction to Partial Differential Equations, Prentice Hall of India, 1995.

.

MATH-202: FUNCTIONAL ANALYSIS

Number of Credits: 4

Prerequisites: This Course assumes basic knowledge in Real Analysis, Linear Algebra and Metric Topology. Knowledge of Integration of a Real Valued Function on a subset of R is desirable. This course is a prerequisite for a course on Advanced Functional Analysis, Integral Equations, Methods of Applied Mathematics.

1) Preliminaries: Metric Space, Metrics on Rn, Holders Inequality, Cauchy-Shwarz Inequality, Minkowskii's Inequality,Banach Spaces: Normed Linear Spaces, Infinite Dimensional Vector Spaces, lp, C(X) for a Compact Hausdorff Space, Cn(U) for a subset of U of Rn, Lp Spaces, Finite Dimensional Normed Linear Spaces**, (15 hours)**

2) Equivalence of Norms on a Finite DimensionalVector Space, Compactness and Finite Dimension, Linear Operators, Boundedness and Continuity, Linear Functional, Algebraic and Topological Dual of a Normed Linear Space. **(15 hours)**

3) Hilbert Spaces: Inner Product Spaces and Hilbert Spaces, Orthogonal Compliment and Direct Sum, Series Related to Orthonormal Sequences, Total Orthonormal Sequences, Legendre, Hermite, Leguerre Polynomials, Representation of Functionals on Hilbert Spaces, Self-Adjoint, Unitary and Normal Operators**. (15 hours)**

4) Fundamental Theorems for Normed Linear Spaces: Hahn-Banach Theorem and Applications to Existence of Functionals, Adjoint Operators, Reflexivity of Spaces, Baire Category Theorem, Uniform Boundedness Theorem. **(15 hours)**

References

[1] Ervin Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, 1978.
[2] Balmohan V Limaye, Functional Analysis,

MATH-203: DIFFERENTIAL GEOMETRY

Number of Credits: 4

Prerequisites: This Course assumes basic knowledge in Real Analysis, Linear Algebra and Several Variable Calculus. This course is a prerequisite for a course on The Theory Lie Groups and a course on Riemannian Geometry.

1) Curves in the Plane and Space: Arc Length, Parametrization, Level sets vs. Parametrized Curves. Curvature: Curvature of Plane curves and Space Curves. Global Properties of Curves: Isoperimetric Inequality and Four-Vertex Theorem. **(15 hours)**

2) Surfaces in Three Dimension: Smooth Surfaces, Tangents Normal and Orientation, Quadric Surfaces, Applications of Inverse Function Theorem. First Fundamental Form Length of Curves on Surfaces, Isometries of Surfaces, Conformal mappings of Surfaces, Surface Area. **(15 hours)**

3) Curvature of Surfaces Second Fundamental Form, Curvature of Curves on a Surface, Normal and Principal Curvature, Geometric Interpretations of Principal Curvatures. Gaussian Curvature: The Gaussian and Mean Curvature, **(15 hours)**

4) The Pseudosphere, Flat Surfaces, Surfaces of constant Curvature, Gaussian Curvature of Compact Surfaces, Gauss map.Geodesics: Definition and Basic Properties, Geodesic Equations, Geodesics on Surface of Revolution, Geodesics as Shortest Paths, Geodesic Coordinates. **(15 hours)**

References

[1] Andrew Pressley, Elementary Differential Geometry, Springer Verlag, 2001.
[2] I. M. Singer and J. A. Thorpe, Lecture Notes on Elementary Topology and Geometry, Springer Verlag.

MATH-204: LIE ALGEBRAS

Number of Credits: 4

Prerequisites: Basic Linear Algebra, basic group theory, basic analysis.

1. Matrices. Real and Complex Matrix Groups. Orthogonal Groups. Topology of Matrix Groups. Tangent space. Definition of Lie Algebras. Some Examples. Subalgebras and Ideals. Homomorphisms. Algebras. Derivations. Structure Constants. Ideals and Homomorphisms. Constructions with Ideals. Quotient Algebras. Correspondence between Ideals. Low-Dimensional Lie Algebras.
(15 hours)

2. Solvable Lie Algebras. Nilpotent Lie Algebras. Subalgebras of gl(V). Nilpotent Maps. Weights. The Invariance Lemma. An Application of the Invariance Lemma. Engel's Theorem and Lie's Theorem. **(15 hours)**

3. Some Representation Theory. Modules for Lie Algebras. Sub-modules and Factor Modules. Irreducible and Indecomposable Modules. Homomorphisms. Schur's Lemma. Representations of sl(2,□). The Modules V_d. Classifying the Irreducible sl(2,□)-Modules. Weyl's Theorem. Cartan's Criteria. Jordan Decomposition. Testing for Solvability. The Killing Form. Testing for Semisimplicity.
(15 hours)

4. Derivations of Semisimple Lie Algebras. The Root Space Decomposition. Cartan Subalgebras. Definition of the Root Space Decomposition. Subalgebras Isomorphic to sl(2,□). Root Strings and Eigenvalues. Cartan Subalgebras as Inner-Product Spaces. Root Systems. First Steps in the Classification. Bases for Root Systems. Cartan Matrices and Dynkin Diagrams. **(15 hours)**

REFERENCES

[1] Karin Erdmann and Mark J. Wildon, *Introduction to Lie Algebras* , Springer Undergraduate Mathematics Series, Springer-Verlag. **2006**.

[2] Kristopher Tapp, *Matrix Groups for Undergraduates*, American Mathematical Society, **2005**.

[3] J.E. Humphreys, *Introduction to Lie algebras and representation theory*, Graduate Text in Mathematics, Springer-Verlag.

[4] N. Jacobson, *Lie Algebras*, Dover Publications.

[5] J.-P. Serre, *Complex Semisimple Lie Algebras*, Springer.

MATH-205: REPRESENTATIONS OF FINITE GROUPS

Number of Credits: 4

Prerequisites: The Course assumes basic knowledge in Groups Theory, Field Theory and Linear Algebra. Knowledge of basic concepts in Module Theory is desirable, but not essential. This course should be offered only for students who have undergone basic courses in Groups, Rings, Fields and Linear Algebra. This will be a prerequisites for a course in Linear Representation of Lie Groups.

1) Representations of Finite Groups: Group Algebra, SimpleModules,Representation, characters of Representations, Orthogaonality Relations, Modular Representations, Integrality of Complex Representations, Burnside's paqb-theorem, Tensor Product of Representations. **(15 hours)**

2) Induced Representations: Induced Representations, Frobenius Reciprocity, Conjugate Representations, Clifford Decomposition, Mackey's Irreducibility Criterion, Wigner-Mackey method of Little Groups. **(15 hours)**

3) Symmetric and Alternating Groups: The Symmetric Groups, Sn, Conjugacy classes in Sn, Irreducible Characters of Sn, Young Diagrams and Young Tableaux, **(15 hours)**

4) Frobenius Young Module for Sn, Specht module for Sn, Standard Y9ung Tableaux, and Basis Theorem for Specht Modules for Sn. Representations of Alternating Groups: Conjugacy Classes in An, Irreducible Representations of An. **(15 hours)**

References:

[1] C. Musili, Representations of Finite Groups, Hindustan Book Agency, 1992.
[2] J. -P. Serre, Linear Representations of Finite Groups, Springer Verlag, 1985.
[3] William Fulton and Joe Harris, Representation Theory, Springer Verlag, 1991.
[4] S. Lang, Algebra, Addison Wesley, 1985.

MATH-206: COMMUTATIVE ALGEBRA

Number of Credits: 4

Prerequisites: This course assumes basic knowledge in Group Theory and Ring Theory. This course is also a prerequisite for a course in Algebraic Topology and Algebraic Geometry.

1) Modules: Definition, Direct sum of Modules, Free Modules and Vector spaces, Homomorphisms and Quotient modules, Simple modules, Modules over PID's. **(15 hours)**

2) Modules with Chain Conditions: Artinian Modules and Rings, Noetherian Modules and Rings, Nil Radical and Jacobson Radical, Radical of an Artinian Ring. **(15 hours)**

3) Homological Algebra: Chain Complexes, Exact Sequences, Five lemma and Snake lemma, Homology Group of a chain complex, Long exact sequence associate with and exact sequence of Chain Complexes. **(15 hours)**

4) Categories an Functors: Definition of Categories and Functors, Examples, Products and Coproducts and their examples, Some Free Objects. **(15 hours)**

References

[1] C. Musili, Introductions to Rings and Modules, Norosa Publishing House, 19912.
[2] S. Lang, Algebra, Addison Wesley, 1985.
[3] J.B. Fraleigh, A First Course in Abstract Algebra, Seventh Edition, Pearson International,2002.

MATH-207: ADVANCED ALGEBRA

Number of Credits: 4

Prerequisites: Basic Group theory and basic Ring theory. This course is also a prerequisite for courses such as Commutative Algebra, Algebraic Geometry, Advanced Number Theory, and Cryptography.

1) Preliminaries: Basic Group Theory, Basic Ring theory and in particular the the Characteristic of an Integral Domain and polynomial Rings over fields and construction of fields using irreducible polynomials, and Basic Linear Algebra including basis and dimension of a Linear Spaces, and Linear maps. **(15 hours)**

2) Extension Fields: Algebraic Extension, Simple Extension, Finite Extension, Algebraic Closure, Geometric Construction, Finite Fields, Primitive Element Theorem. **(15 hours)**

3) Galois Theory: Automorphisms of Fields, The Isomorphism Extension Theorem, Splitting Fields, Separable Extensions, Normal Series, Jordan Holder Theorem, **(15 hours)**

4) Solvability of Groups, Galois Theory, Illustrations of Galois Theory, Cyclotomic Extensions, Insolvability of Quintics. **(15 hours)**

References

[1] J.B. Fraleigh, A First Course in Abstract Algebra, Seventh Edition, Pearson International.
[2] I. N Herstein, Topics in Linear Algebra, Second Edition, Wiely Student Edition, 2006.

MATH-208: INTEGRAL EQUATIONS

Number of Credits: 4

Prerequisites Real Analysis,Linear Algebra, Differential equations, Calculus of Several variables.

1) Classification, Integral Equations with Separable Kernels, Method of Successive Approximations, Resolvent Kernel and its Properties, Classical Fredholm Theory, Symmetric Kernels, Hilbert-Schmidt Theory **(15 hours)**
2) Rayleigh-Ritz Method, Singular Integral Equations, Abel and Cauchy Type and Hilbert Kernel, Integral Transform Methods (Laplace, Fourier and Hilbert) **(15 hours)**
3) Applications to Ordinary Differential Equations, Initial Value Problems and Boundary Value Problems, Green's Functions, Decomposition methods,**(15 hours)**
4) Non linear Fredholm Integral Equations and Volterra Integral Equations, Existence and uniqueness of solutions.**(15 hours)**

References

[1] (Principal Text) Ram P Kanval, Linear Integral Equations, Theory and Techniques.
[2] Courant and Hilbert, Methods of Mathematical Physics, Vol.I.
[3] S.G.Mikhilin, Integral Equations.
[4] I.G.Petrovsky, Lectures on the theory of Integral equations.
[5] K.Yoshida, Lectures on Differential and Integral Equations.

MATH-301: TOPOLOGY II

Number of Credits: 4

Prerequisites: This Course assumes basic knowledge in General Topology and Basic Groups Theory. Knowledge of Commutative Algebra is desirable. This course is a desirable prerequisite for an advanced course in Algebraic Topology.

1) Classification of Surfaces: Cutting and Pasting, The Classification Theorem, Construction of Compact Surfaces.
The Fundamental Group: Homotopy of Paths, The Fundamental Groups, Covering Spaces, The Fundamental Groups of the Circle, Retractions and Fixed Points, Fundamental Theorem of Algebra, Borsuk-Ulam Theorem, Deformation Retracts and Homotopy Types, Fundamental Group of Sn, Fundamental Group of Some Surfaces. **(15 hours)**

2) Separation Theorems in the Plane: Jordan Separation Theorem, Invariance of Domain, Jordan Curve Theorem, Winding Number of a Simple Closed Curve, Cauchy's Integral Formula. **(15 hours)**

3) Seifert van-Kampen Theorem: Free Group, Free Products of Groups, Statement of Seifert van-Kampen Theorem, Fundamental Group ofWedge, Fundamental Group of Surfaces. **(15 hours)**

4) Classification of Covering Spaces: Equivalence of Cover9ng Spaces, The Universal Covering Space, Covering Transformations, Existence of Covering Spaces.
Application to Group Theory: Covering Spaces of a Graph, The Fundamental Group of a Graph, Subgroups of a Free Group is a Free Group. **(15 hours)**

References

[1] James Munkres, Topology, Second Edition, Pearson Education Asia, 2000.
[2] W. S Massey, Algebraic Topology, Springer Verlag.

MATH-302: MEASURE THEORY

Number of Credits: 4

Prerequisites: Real Analysis including Riemann Integration, First Course in Topology, Basic Functional Analysis.

1) Lebesgue Measure: Outer Measure, Measurable Sets and Lebesgue Measure, Measurable Functions, Egoroff's Theorem.
Lebesgue Integral: Motivation and Definition for Simple Functions, Bounded Measurable Functions, General Measurable Functions and Properties**, (15 hours)**

2) Bounded Convergence Theorem, Fatous Lemma, Monotone Convergence Theorem Lebesgue Convergence Theorem. **(15 hours)**

3) The Classical Banach Spaces: The Lp-Spaces, The Minkowski and Holders Inequality, Convergence and Completeness, The Dual of Lp-spaces. GeneralMeasures: Measure Space, Finiteness and _-finiteness, Measurable Functions, **(15 hours)**

4) Integration, Outer Measure and Measurability, A brief Discussion (without proof) on Extension Theorem, Baire sets and Borel sets, The Regularity of Baire and Borel measures, The Construction of Product Measures, Fubini's Theorem**(15 hours)**
.

References

[1] H.L. Royden, Real Analysis, Third Edition Prentice Hall India, 1988.
[2] Walter Rudin, Real and Complex Analysis, Third Edition, McGraw-Hill International Editions, 1987.
[3] Carothers, Real Analysis, Springer Verlag.

MATH-303: DIFFERENTIAL TOPOLOGY

Number of Credits: 4

Prerequisites: This Course assumes basic knowledge in Real Analysis, Linear Algebra, Topology and Several Variable Calculus. This course is a prerequisite for a course on The Theory Lie Groups and a course on Riemannian Geometry.

1) Manifolds and Maps: Submanifolds of Rn+k, Differential Structure, Differentiable maps and Tangent Bundle, Embeddings and Immersions, Manifolds with Boundary. **(15 hours)**

2) Function Spaces: The Weak and Strong Topologies on Cr(M,N), Approximations, Approximation on @-manifolds and manifold Pairs, Jets and the Baire property. Transversality: The Morse-Sard Theorem, Transversality. **(15 hours)**

3) Vector Bundles and Tubular Neighborhoods: Vector Bundles, Construction with vector bundles, The Classification o f Vector Bundles, Oriented Vector Bundles, Tubular Neighborhoods, **(15 hours)**

4) Collars and Tubular Neighborhoods of neat submanifolds, Analytic Differential Structure. Degree Intersection numbers and Euler characteristic: Degree of a Map, Intersection numbers and Euler Characteristic. **(15 hours)**

References

[1] Morris W. Hirsch, Differential Topology, Springer Verlag, 1976.
[2] John M. Leee, EIntroduction to Smooth Manifiolds, Springer Verlag.

MATH-304: STURM-LIOUVILIE PROBLEMS

Number of Credits: 4

Prerequisites: Real Analysis,Advanced calculus,Differential equations and Methods of applied mathematics.

1) Review of Linear Ordinary Differential Equations, Principle of Superposition, Boundary Conditions, Adjoint Equation. Green's Formulae, **(15 hours)**

2) Vibrating String, Sturm-Liouvilie Problems, Singular Boundary Points**, (15 hours)**
3) Asymptotic Behaviour, Eigen Value Problem with Continuous Spectra**,(15 hours)**
4) Vibrating String and Suspended Rope, Associated Integral Equation.**(15 hours)**

References

[1] M. P. S. Esthan, Theory of Ordinary Differebntial Equations, Van Nostrand, 1970
[2] (Reference Text) R. Courant, D Hilbert, A Methods of Mathematical Physics, vol. I,Wiley Eastern, New Delhi, 1975
[3] Coddington Levigson

MATH-401: INTRODUCTION TO LIE GROUPS

Number of Credits: 4

Prerequisites: The Course assumes basic knowledge in Groups Theory, Linear Algebra, Several Variable Calculus, Differential Geometry and Topology.

1) Matrix Lie Groups: Definition of Matrix Lie Groups and Examples, Compactness and Connectedness and Simple Connectedness. Homomorphisms and Isomorphisms, Polar Decomposition of SL(n,R) and SL(n,C), General Definition of Lie Groups. **(15 hours)**

2) Lie Algebras and Exponential Mapping: Definition and some properties, Matrix Logarithm, Lie Algebra of a matrix Lie Group. Properties of Lie Algebras, Exponential Mapping. Complexification of Real Linear Lie Algebra. **(15 hours)**

3) Baker Campbell-Hausdorff formula: BCH Formula for Heissenberg Group, General BCH Formula, derivative of Exponential Mapping, Series form of BCH Formula, Group versus Lie Algebra Homomorphisms, Covering Groups, Subgroups and Subalgebras. **(15 hours)**

4) Basic Representation Theory: Definition of Representations and Examples, Unitary Representations of Heissenberg Groups, Irreducible Representations of su(2), Direct Sums of Representations, Tensor Product of Representations, Dual Representations, Schur's Lemma, Group versus Lie Algebra Representations, Complete Reducibility. **(15 hours)**

References

[1] Brian Hall, Lie Groups Lie Algebras and Representations, An Elementary Introduction, Springer Verlag, 2003.

MATH-402: MATHEMATICAL MODELLING

Number of Credits: 4

Prerequisites, Real Analysis,Advanced calculus, Differential equations, Difference equations, Statistics,

1) Introduction, Classification, Techniques and Examples,Modelling Change, Modelling process with proportionality and geometric similarity, Mathematical Modelling through Ordinary differential equations, **(15 hours)**

2) Ordinary Differential Equations of first order and of second order, First order systems of Ordinary Differential Equations, **(15 hours**)

3) Mathematical Modelling through Partial differential equations, (**15 hours)**

4) Some equations case studies, Specific examples to be included, Model fitting and experimental modelling, Case studies. **(15 hours)**

References

[1] J.N.Kapur, AMathematical Modelling, Wiley Eastern ltd.
[2] F.Chorlten, Differential and difference equations., von Nostqand.
[3] F.R.Giordano, M.D.Weir, W.P.Fox, A first course in Mathematical modeling, Thomson Publications.
[4] D.N.Burghes, Modelling with Differential Equations, Ellis Horwood and John Wiley.
[5] J. Sandefur, Elementary Mathematical Modeling, Thomson Publications.

<u>MATH-403: DYNAMICAL SYSTEMS</u>

Numbers of Credits-4

<u>Prerequisites</u>:Basic Analysis, ODE. Basics of SVC

1. Reviews of First order linear systems of differential equations, eigen values, diagonalisation and Jordan canonical form of matrices. Nonlinear systems: Local Theory Fundamental concepts, existence and uniqueness theorem Maximal interval of existence,
   (15 hours)
2. Flow defined by a differential equation and linearization, Stable manifold and Hartman-Grobman theorem **.** Stability and Liapunov function Saddles, nodes, Foel and centers Non-hyperbolic critical points in R2, Grandient and Hamiltonian systems. **(15 hours)**
3. Nonlinear systems: Global Theory Global existence theorems, Limit sets and attractors, Periodic orbits, Limit cycles and Separatrix cycles,
   (15 hours)
4. Poincare map and stable manifold theorem for periodic orbits, Hamiltonian systems with two degrees of freedom Poincare- Benedixon theory in R2, Lienard systems, Poincare sphere and the Bheavior at infinity, Global Phase Portraits and Separtrix Configurations.
   (15 hours)

<u>References</u>

1) Lawrence Perko, Differential Equations and Dynamical Systems, (Section 1.1-3/11). Text in Applied Mathematics series, Springer-Verlag, 1991.
2) Moriss W. Hirsch, Stephane Smale and Robert L. Devaney, Differential Equations, Dynamical systems, and an introduction to Chaos (Chapter 1-10,17), Academic Press,2004

MATH- 001 : Statistics

[Problems oriented for Students of PG courses other than M.Sc. (Mathematics) M.Sc. (Physics), M.Sc. (Electronics) and MCA]

Number of Credits :4

Prerequisites: Basic Mathematics

1) Review of basics. Set theory and Calculus.

Introduction and Data Presentation .Concepts of homogeneity and variation, population and sample, parameters and statistics probability. Types of data: nominal, ordinal data, ranked data, numerical discrete data and numerical continuous data. Tables and graphs: frequency distributions, relative frequency; bar charts, histograms, scatter plots, box plots, line graphs, pie graphs. Measures of central tendency: mean, median, mode, percentile . Measures of dispersion: range, inter-quarter range, variance, standard deviation, coefficient of variation. **(15 hours)**

2) Probability. Basic results (without proof). Random variable. Discrete and Continuous random variables. Expectation and Variance. Discrete and continuous probability distributions. Special distributions like Binomial, Poisson, Normal, Uniform Gamma, Exponential and Chi square. Correlation and Regression. Coefficient of determination. Prediction. **(15 hours)**

3) Sampling. Sampling distribution of the mean, sampling error of the mean. Principles of Statistical Inference. Confidence interval. Hypothesis test: concepts of hypothesis test, type I error, type II error, power. Comparison of Two Means. One sample test. Paired samples test. Independent two samples test: equal variances, unequal variances. Z- test, t-test and F test. **(15 hours)**

4) Significance of correlation and regression coefficients. ANOVA. Non Parametric tests. Sign test The sign test . Wilcoxon signed-rank test. Wilcoxon rank sum test.Multiple regression. Related concepts: the coefficient of multiple regression, the coefficient of multiple correlation, the coefficient of partial correlation, the coefficient of determination. **(15 hours)**

Reference:

1. Mathematical Statistics: Gupta and Kapur, S.Chand
2. Mathematical Statistics with applications: Miller and Miller, Pearsons.
3. Fundamentals of Biostatistics ,Bernard Rosener , academic Internet Publishers