Change of Discipline Test Syllabus for MSc Mathematics

Algebra

1. Groups definition and elementary properties; Finite group and subgroups; Examples; Cyclic groups; Properties of cyclic groups; Classification of subgroups of cyclic groups.

2. Permutation groups; Cycle notation; Properties of permutations. Isomorphisms: Definitions and examples; Cayley's Theorem; Properties of isomorphisms; Automorphism.

3. Cosets; Properties of cosets; Lagrange's Theorem and consequences; An application of cosets to permutation group.

4. Definition and examples of external direct product; Properties of external direct product; The group of units modulo n as an external direct product.

5. Normal subgroups and factor groups; Application of factor groups; Internal direct product. Definition and examples of group homomorphisms; Properties of homomorphisms; First Isomorphism Theorem.

6. Fundamental Theorem of Finite Abelian Groups; Isomorphism classes of abelian groups; Proof of Fundamental Theorem.

7. Rings; Properties of rings; Subrings; Integral domains; Examples of integral domains; Fields; Charecteristic of a ring.

8. Ideals and Factor rings; Prime ideals; maximal ideals; Ring homomorphisms; Properties of ring homomorphisms; Field of quotients.

9. Polynomial rings; The Division Algorithm and consequences.

PRINCIPAL TEXT:
Joseph A. Gallian, *Contemporary Abstract Algebra*, (8^{th} ed), Cengage Learning

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Analysis

1. Improper Integrals: Improper Integrals of type I; Cauchy's general principle of convergence for Improper integrals of type I; Comparison test for improper integrals of type I; Comparison test in limit form for improper integrals of type I; p - test for improper integrals of type I; Improper Integrals of type II; Cauchy's general principle of convergence for Improper integrals of type II; Comparison test for improper integrals of type II; Comparison test in limit form for improper integrals of type II; p - test for improper integrals of type II; Improper Integrals of type III.

2. Beta and Gamma Functions: Definitions of Beta and Gamma Functions and their convergence. Properties of Beta and Gamma functions. Relation between beta and Gamma functions. Legendre's duplication formula.

3. Power series in IR: Definition and examples. Radius and interval of convergence, Uniform convergence and absolute convergence, Term by term differentiation and integration of power series in IR. Power series definitions of Exponential, Logarithmic and trigonometric functions, their properties. Weierstrass' polynomial approximation theorem. [Statement only and explanation]

4. Inner product spaces: Square integrable functions. Usual integral inner product on C[a, b] and its properties. Norm induced by usual integral inner product. Orthogonal and orthonormal sequences of functions in C[a, b] with usual integral inner product. Complete orthogonal and orthonormal set in C[a, b] with respect to usual integral inner product. Bessel's inequality and Parsevel's identity set in C[a, b] with respect to usual integral inner product.

5. Fourier series: Fourier series of real functions on $(-\pi, \pi)$ and $(0, \pi)$. Fourier coefficients; properties of Fourier coefficients; the Fourier series of a function relative to an orthonormal system. Bessel's inequality. Trigonometric Fourier series, Fourier series of odd & even function. Integration & differentiation of Fourier series at a point. Fourier theorem. Fourier Series of real functions on $(c, c+2l)$. Riemann-Lebesgue Lemma. Parsevel's identity.

Principal Texts :

1. S. C. Malik : Principles of Mathematical Analysis
2. R.D.Bhat : Mathematical analysis II
3. B.S.Grewal: Higher Engineering Mathematics

Foundations of Mathematics

1. **Statements and Logic**: Statements; Statements with quantifiers; Compound statements; Implications; Proofs in Mathematics.
2. **Sets**: Sets; Operations on sets; Family of sets; Power sets; Cartesian product of sets.

3. **Functions**: Basic definitions; one-one, onto functions and bijections; Composition of functions; Inverse of a function; Image of subsets under functions; Inverse image of subsets under functions.

4. **Relations**: Relation on sets; Types of relations; Equivelence relations; Equivelence classes and partitions of sets.

5. **Induction Principles**: The induction Principle; The Strong Induction Principle; The Well-Ordering Principle; Equivalence of the three principles.

6. **Countability of sets**: Sets with same cardinality; Finite sets; Countable sets; Comparing cardinality.

7. **Order Relation**: Partial and total orders; Chains, bounds and maximal elements; Axiom of choice and its equivalents.

PRINCIPAL TEXT:

1. Kumar, S. Kumaresan and B.K. Sarma, *A Foundation Course in Mathematics*, Narosa Publisher, 2018

Differential Equations

1. **<u>Review of First and Second order ordinary differential equations:</u>**

Basic concepts . First order ordinary differential equations with constant coefficients. Homogeneous and non homogeneous equations. Exact and non exact differential equations. Integrating factors. Second order differential equations with constant coefficients. Complementary function and particular solution. Use of known solution to find linearly independent second solution. Method of variation of parameters. Equations with variable coefficients. Method of undetermined coefficients.

2. **Power Series Solutions of Some Linear Equations:**

   Homogeneous equations with analytic coefficients. Legendre equation, Justification of power series method, Introduction to linear equations with Regular singular points, Euler equation, example and general case of second order equations with regular singular points, A convergence proof, Exceptional cases, Bessel equation, Regular singular points at infinity. Properties of Legendre Polynomials and Bessels function. Generating function.

3. **Laplace Transforms:**

   Laplace transforms of various functions, General properties of Laplace transforms, Inverse Laplace transforms, Convolution theorem, Application of Laplace transforms to solve differential equations.

4. **Numerical Methods of Solving Differential Equations:**

   Picard’s method, Euler’s method, Modified Euler’s method, Runge-Kutta method, Milne’s method, Adams-Bashforth-Moulton method.

PRINCIPAL TEXTS:

1) An Introduction to Ordinary Differential Equations by Earl A. Coddington, Prentice-Hall of India Private Limited, New Delhi.
2) A text book of Differential Equations by S.G.Deo, V Raghavendra and V. Lakshmikantham. TMH edition.
3) Numerical methods by M.KJain, S.R.K.Iyenger, R. K..Jain

Complex Analysis

1. **Complex Numbers:**
   Sums and products, Algebraic properties, Vectors and moduli, Complex conjugates, Exponential form, Arguments of products and quotients, Roots of complex numbers, Regions in the complex plane.

2. **Analytic Functions:**
   Functions of complex variable, Limits, Theorems on limits, Continuity, Derivatives, Differentiation formulas, Cauchy-Riemann equations, Sufficient condition for Differentiability, Polar coordinates, Analytic functions, Harmonic functions.

3. **Elementary Functions:**
   Exponential function, Logarithmic function, Branches and Derivatives of Logarithms, Identities involving logarithms, Complex exponents, Trigonometric functions, Hyperbolic functions, Inverse trigonometric and hyperbolic functions.

4. **Integrals:**

Derivatives of functions, Definite integrals of functions, Contours, Contour integrals, Contour integrals of functions with branch cuts, Upper bounds for moduli of contour integrals, Antiderivatives, Cauchy-Goursat theorem (without proof), Simply and Multiply connected domains, Cauchy integral formula and extension of Cauchy integral formula, Liouville's theorem, Fundamental theorem of algebra, Maximum modulus principle.

5. **Series:**
   Convergence of sequences and series, Taylor series, Taylor's theorem, Laurent series, Laurent's theorem.[statements only and applications]

6. **Residues and Poles:**
   Isolated singular points, Residues, Cauchy Residue theorem, Residue at infinity, The three types of Isolated singular points, Residues at poles, Zeros of analytic functions, Zeros and Poles.

7. **Mappings by Elementary functions:**
   Fractional Linear transformations, Transformation w=1/z, Mappings by 1/z, Mobius transformation.

PRINCIPAL TEXT:

Complex Variables and Applications by James Brown and Ruel Churchill, Eighth Edition, McGraw-Hill International Edition. (With omission of sections 13, 14,27,28,47,63,64,65,66,67,80,81,82,83,84,85,88,89 of the principal text.)

Metric spaces

1. **INTRODUCTORY CONCEPTS IN METRIC SPACES:-**
   Definition and Examples of Metric Spaces, Open Balls and Closed Balls, Hausdorff Property, Interior Point and Interior of a Set, Open Sets and their properties, Closed Sets and their properties, Limit Points and Isolated Points, Derived Set and its properties, Closure of a Set and its properties, Boundary Points, Distance between Sets, Diameter of a Set, Subspace of Metric Space and its properties, Boundedness in a Metric Space.

2. **COMPLETENESS IN METRIC SPACES:-**
   Sequence in a metric Space, Convergence of a Sequence in a Metric Space, Cauchy Sequence in a Metric Space, Complete Metric Spaces, Cantor's Intersection Theorem, Dense Sets.

3. **CONTINUOUS FUNCTIONS ON METRIC SPACES:-**
   Sequential Continuity, Continuity of Functions using Open Sets and Closed Sets, Continuity of Functions using Closure of a Set, Contraction map and its properties, Fixed Points, Picard's Fixed Point Theorem, Picard's Existence and Uniqueness Theorem for First Order Initial Value Problem.

4. **COMPACTNESS IN METRIC SPACES:-**
   Compact Metric Spaces and Compact Sets, Examples of Compact Metric Spaces and Compact Sets, Properties of Compact Metric Spaces and Compact Sets, Sequential Compactness, Bolzano – Weierstrass Property, Heine – Borel Theorem, Totally Boundedness, Equivalence of Compactness and Sequential Compactness, Lebesgue Covering Lemma, Compactness and Finite Intersection Property, Continuous Functions and Compactness.

5. **CONNECTEDNESS IN METRIC SPACES:-**
   Separated Sets, Connected Metric Spaces and Connected Sets, Properties of Connected Metric Spaces and Connected Sets, Connected Subsets of IR, Connectedness and Continuous Functions, Intermediate value Theorem.

Principal text:

Mathematical Analysis-I (Metric Spaces) : J.N.Sharma